

# Variable Demand Elasticity, Markups, and Pass-Through\*

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PRELIMINARY AND INCOMPLETE

## **Abstract**

In these notes we analyze the properties of a number of demand functions that have been used in international macroeconomic and trade models as alternatives to CES. These demand functions generate variable markups by means of variable elasticity of demand, and yield closed-form relationships between markups and prices. We exploit the tractability of the demand systems to discuss the pass-through of cost-shocks into prices and the distribution of markups and pass-through in the economy.

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# 1 Introduction

Understanding the link between movements in international prices or markups and aggregate shocks, such as trade liberalizations or exchange-rate fluctuations, is one of the key challenges for international economists. In recent years, a growing literature has tried to learn more about this link by studying firm behavior at the micro-economic level, thanks to the increased availability of micro data on firms and international trade<sup>1</sup>. One of the most robust findings of this literature is that heterogeneous firms charge heterogeneous markups, and only respond partially to exchange rate shocks, i.e. the so-called cost-shock *pass-through* into firm prices is incomplete (e.g. [Atkeson and Burstein \(2008\)](#); [Goldberg and Hellerstein \(2013\)](#); [Amiti et al. \(2014\)](#)). These and other aspects of the data are hard to reconcile within a unified - yet tractable - theoretical framework. Paradoxically, the workhorse model in international economics still assumes CES demand and monopolistic competition (CES+MC), which together imply constant markups and complete pass-through in equilibrium. Furthermore, the assumption of constant demand elasticity excludes *a priori* any welfare effects of international shocks that can derive from movements of profit margins<sup>2</sup>. It goes without saying that for many economic questions involving pricing and welfare analysis the CES+MC model has little merit, even as a first approximation.

In these notes we analyze the properties of a number of demand functions that have been used in international macroeconomic and trade models as alternatives to CES. The demand systems we consider feature variable markups by means of variable demand elasticity - as in the tradition of [Dornbusch et al. \(1987\)](#) - and are tractable enough to yield closed-form expressions for markups and prices<sup>3</sup>. We first show that for a general class of demand functions, we can write the outcome variables of interest (i.e. markups and pass-through

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<sup>1</sup>See [Burstein and Gopinath \(2014\)](#) for a recent survey of the empirical and theoretical literature.

<sup>2</sup>In fact, [Arkolakis et al. \(2015\)](#) show that welfare gains from trade liberalization *significantly* change depending on whether or not one restricts markups to be constant across firms.

<sup>3</sup>Our approach is inspired by [Burstein and Gopinath \(2014\)](#). While they review different modeling approaches to generate variable markups, we focus on the variable elasticity of demand channel, generalizing the discussion and characterizing analytically the implications for cost-shock pass-through.

elasticities) as functions of one sufficient statistic only, namely the ratio of firm price to the market *choke price*. We then consider different examples of demand functions in this class, and study the predictions for markups and pass-through distribution in the economy. We believe that studying the properties of different demand functions is important for two reasons. First, it provides guidance for structural models, which are needed whenever one wants to speak to welfare or perform counterfactual analysis. Any structural model requires parametric restrictions on consumer demand, market structure, or the nature of competition, which is *not* without loss of generality. Second, in the class of models we study, results about markups and pass-through of shocks hinge *exclusively* on the assumptions on the demand side of the economy. A careful selection of the structure of demand is thus a necessary condition for making the model empirically relevant<sup>4</sup>.

The notes are structured as follows. In Section 2, we lay out a general framework encompassing the most prominent alternatives to CES demand, building on [Arkolakis et al. \(2015\)](#). We then move on to study several examples of utility (and demand) functions which fall into this class, and discuss their main properties. In Section 3, we show how these results can be useful in determining the model fit to the data. To this end, we also review some relevant empirical evidence on markups and international prices in section 3.2, together with some limitations of this class of models in section 3.4. Section 4 discusses the [Atkeson and Burstein \(2008\)](#) model of oligopolistic competition and CES demand, along with other models of variable markups considered in the literature, which do not directly fit into the framework presented in the main part of the text. Section 5 concludes.

## 2 Models with Variable Elasticity of Demand

The building block of our discussion is a standard model of international economics. We let  $\omega \in \Omega$  denote both a variety of the consumption good in country  $j$ , and the firm producing that variety. We denote equilibrium price and quantity of variety  $\omega$  as  $p_j(\omega)$  and  $q_j(\omega)$ ,

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<sup>4</sup>See also [Mrázová and Neary \(2013\)](#); [Zhelobodko et al. \(2012\)](#) for similar discussions.

respectively. In principle, the good could be sold in the domestic or foreign market, and the subscript  $j$  refers to the country where the good is sold and consumed. In what follows, we will omit the country subscript unless necessary for the discussion.

## 2.1 The General Framework

All consumers have the same preferences. Demand for good  $\omega$ , when income is  $w$ , and prices  $\mathbf{p} \equiv \{p_\omega\}_{\omega \in \Omega}$ , is

$$q_\omega(p_\omega, P, Q) = QD(p_\omega/P), \quad (1)$$

where we assume that  $D(x) \in \mathcal{C}^2(x)$  is twice continuously differentiable, with  $D'_x < 0$ . The aggregate demand shifters  $Q(\mathbf{p}, w)$  and  $P(\mathbf{p}, w)$  are jointly determined from<sup>5</sup>

$$\begin{aligned} \int_{\omega \in \Omega} \left[ H\left(\frac{p_\omega}{P}\right) \right]^\beta \left[ p_\omega QD\left(\frac{p_\omega}{P}\right) \right]^{1-\beta} d\omega &= w^{1-\beta} \\ Q^{1-\beta} \left[ \int_{v=1}^\infty p_\omega QD\left(\frac{p_\omega}{P}\right) \right]^\beta d\omega &= w^\beta, \end{aligned} \quad (2)$$

with  $\beta \in \{0, 1\}$  and  $H(\cdot)$  strictly increasing and concave. For a complete description of this demand system the reader should refer to [Arkolakis et al. \(2015\)](#). Here we just want to remark that this framework encompasses both homothetic and non homothetic consumer preferences. If  $\beta = 1$ ,  $P(p, w)$  is independent of  $w$  and that  $Q(p, w)$  is proportional to  $w$ , which means that preferences are homothetic. Conversely, if  $\beta = 0$ , one can show that preferences are non-homothetic. Below, we will discuss examples for both demand types. We make the following assumption:

**A1**      **[Choke Price]** *There exists a price  $p^* \in \mathbb{R}$  such that for all  $p \geq p^*$ ,  $D(p/P) = 0$ .*

The choke price is common across varieties due to symmetric preferences, and can be interpreted as the maximum price consumers are willing to pay to buy any variety  $\omega \in \Omega$  in positive amounts, i.e. a “reservation” price. In the CES case this reservation price is infinite<sup>6</sup>.

<sup>5</sup>Note from (1) that  $q_\omega(p_\omega, P, Q) = q(p_\omega/P; Q)$ , which means that while the aggregate shifter  $Q$  affects only the level of demand,  $P$  affects both the level and the curvature of demand.

<sup>6</sup>The existence of a choke price implies that there might be goods there are available for consumption, but not consumed in equilibrium. Moreover, the set of available goods might change endogenously in equilibrium.

**Supply Side** Consider the optimization problem of firm  $\omega$ , and let  $c(\omega; \vartheta)$  denote its marginal cost. Parameter  $\vartheta$  denotes cost-shifters, such as transportation costs or bilateral exchange rates. We assume monopolistic competition and segmented good markets, which means that the firm chooses price  $p$  in order to maximize

$$\pi(c, Q, P) = \max_p \{(p - c)q(p, P, Q)\} \quad (3)$$

taking demand and (market-specific) aggregate variables  $Q$  and  $P$  as given.

**Equilibrium** Let  $\varepsilon(\omega) \equiv -\frac{\partial \log q(\omega)}{\partial \log p(\omega)}$  be the price partial elasticity of demand. As it is standard, we can write the equilibrium price as a markup times marginal costs:

$$p(\omega) = \frac{\varepsilon(\omega)}{\varepsilon(\omega) - 1} \cdot c(\omega; \vartheta), \quad (4)$$

where markup  $\mu(\omega) \equiv \frac{\varepsilon(\omega)}{\varepsilon(\omega) - 1}$  is a function of the demand elasticity, and thus depends on demand side conditions. Note that this formulation implies that variable markups are achieved by means of variable elasticity of demand. Since demand in (1) is just a function of the relative price, it follows that both elasticity and markups can be written as functions of  $\frac{p(\omega)}{P}$ , i.e.  $\varepsilon(\omega) = \varepsilon(p(\omega)/P)$  and  $\mu(\omega) = \mu(p(\omega)/P)$ . We are now interested in studying cost-shock pass-through, i.e. the price elasticity to an exogenous shift in  $\vartheta$ . We first rewrite (log) firm price as

$$\log p(\omega) = \log \mu(y(\omega)) + \log c(\omega; \vartheta), \quad (5)$$

where  $y(\omega) \equiv \frac{p(\omega)}{P}$ . We also denote with  $\Gamma(\omega) \equiv -\frac{d \log \mu(\omega)}{d \log y(\omega)}$  the markup elasticity with respect to the relative price. In the CES case, markups are constant and  $\Gamma(\omega) = 0$ . We take log differences and write

$$d \log p = -\Gamma(d \log p - d \log P) + \rho_\vartheta d \log \vartheta, \quad (6)$$

where  $\rho_\vartheta \equiv \frac{\partial \log c}{\partial \log \vartheta}$ , is the partial-elasticity of the marginal cost to a shock to  $\vartheta$ , e.g. to an exchange-rate shock. We assume for simplicity that  $\rho_\vartheta = 1$ , which means that the pass-through of the exogenous shock into marginal costs is *complete*<sup>7</sup>. We can then rewrite (6) as

$$\frac{d \log p}{d \log \vartheta} = \frac{1}{1 + \Gamma} - \frac{\Gamma}{1 + \Gamma} \frac{d \log P}{d \log \vartheta}. \quad (7)$$

Equation (7) says that movements in  $\vartheta$  will affect prices both directly, and indirectly, through their GE effect on the aggregate price  $P$ . Since our analysis will be mostly partial equilibrium, we focus on the direct effects and define firm pass-through  $\Phi(\omega)$  as the price elasticity of a cost shock controlling for changes in costs and GE effects through  $P$ , i.e

$$\Phi(\omega) \equiv \left. \frac{\partial \log p(\omega)}{\partial \log \vartheta} \right|_P = \frac{1}{1 + \Gamma(\omega)}. \quad (8)$$

Equation (8) says that following a 1% increase in costs, the price increases by  $\Phi(\omega) = \frac{1}{1 + \Gamma}$ . When markups are constant,  $\Gamma = 0$  and so  $\Phi(\omega) = 1$ , i.e. the pass-through is *complete*. When  $\Gamma(\omega) > 0$ , then  $\Phi(\omega) \in (0, 1)$ , and pass-through is said to be *incomplete*. In this case the price moves less than one-to-one with the shock, and it is what we observe in the data. Note that in this simplified framework, incomplete pass-through only depends on variable markups, i.e. by firms moving markups so as to offset changes in marginal costs.

### 2.1.1 A Sufficient Statistics Result

In what follows we restore full notation, and index variables by the country where they are sold. Let  $x_j(\omega) \equiv \frac{p_j(\omega)}{p_j^*}$  denote the relative price of good  $\omega$  in country  $j$ , over the choke price in that market. Using the definition in A1, we can obtain  $p_j^*$  as

$$p_j^* \text{ s.t. } D \left( \frac{p_j^*}{P_j} \right) = 0, \quad (9)$$

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<sup>7</sup>This assumption is commonly - and often implicitly - made in the literature, but it is not without loss of generality. In general, this is equivalent to assuming that one could condition on costs in the empirical analysis, which is a strong assumption given the limited data availability on firm level marginal costs.

which is equivalent to write

$$p_j^* = D_j^{-1}(0)P_j. \quad (10)$$

Equation (10) says that the choke price is a function of the aggregate price index, and demand parameters. Due to symmetric preferences,  $p_j^*$  is common across goods. We can use (10) in (1) and write demand for any variety  $\omega$  as

$$\begin{aligned} q_j(\omega) &= Q_j D \left( \frac{p_j(\omega)}{p_j^*} D_j^{-1}(0) \right) \\ &= Q_j \tilde{D}_j(x_j(\omega)). \end{aligned} \quad (11)$$

We take logs of (11) and write

$$\log q_j(\omega) = \log Q_j + \log \tilde{D}_j(x_j(\omega)), \quad (12)$$

from which it follows immediately that

$$\varepsilon_j(\omega) \equiv -\frac{\partial \log q_j}{\partial \log p_j} = -\frac{\partial \log \tilde{D}_j}{\partial \log p_j} \quad (13)$$

$$= \tilde{\varepsilon}_j(x_j(\omega)), \quad (14)$$

i.e. demand elasticity can be written as a function of  $x_j(\omega)$  only. Similarly, for markups, markup elasticity and pass-through we get that

$$\mu_j(x_j(\omega)) = \frac{\tilde{\varepsilon}_j(x_j(\omega))}{\tilde{\varepsilon}_j(x_j(\omega)) - 1}, \quad (15)$$

$$\Gamma_j(x_j(\omega)) = -\frac{d \log \mu(x_j(\omega))}{d \log x_j(\omega)} \quad (16)$$

and

$$\Phi(x_j(\omega)) = \frac{1}{1 + \Gamma(x_j(\omega))}. \quad (17)$$

Equations (13) to (17) say that knowing the relative price of the firm  $x_j(\omega)$  is *sufficient* to learn about firm's elasticity, markups and pass-through. This sufficient statistic result constitutes the key contribution of our exercise. Later we will show that in a standard GE model in international economics, the sufficient statistic  $x_j(\omega)$  can be mapped one-to-one to firm size.

In the remaining part of the Section, we are going to show how this general framework can be useful to study the properties of specific demand systems. We consider four examples which have been studied in the literature as alternative to CES, i.e. the Pollak's Additive Utility Functions (e.g. [Arkolakis et al. \(2015\)](#)), the Kimball Utility Function (e.g. [Kimball \(1995\)](#); [Klenow and Willis \(2006\)](#); [Gopinath and Itskhoki \(2010\)](#)), Addilog Preferences (e.g. [Bertoletti et al. \(2015\)](#); [Bertoletti and Etro \(2015\)](#)) and Quadratic Mean of Order r (QMOR) preferences ([Feenstra \(2014\)](#)).

## 2.2 Pollak's Additive Utility Functions

We start by considering the demand system studied by [Pollak \(1971\)](#). Preferences are given by an additive utility function of the form

$$\int_{\Omega} \alpha (q_j(\omega) - \gamma)^{1-\frac{1}{\sigma}} \quad (18)$$

with  $q(\omega) > \gamma$  and either (i)  $\sigma > 1$  and  $\alpha > 0$ , or (ii)  $\sigma \in (0, 1)$  and  $\alpha < 0$ . The indifference maps corresponding to (18) are homothetic to the point  $\gamma$  instead of to the origin. The demand function for each good  $\omega$  can be obtained by maximizing (18) subject to a standard budget constraint, and can be written as

$$q_j(\omega) = \gamma + \kappa p_j(\omega)^{-\sigma}, \quad (19)$$



where  $\kappa$  is a function of endogenous variables and parameters, which we write as constant for simplicity. Demand (19) belongs to a class of demand functions which are locally linear in income. We distinguish three relevant cases:

1.  $\gamma = 0$  and  $\kappa, \sigma > 0$  - this corresponds to CES preferences;
2.  $\gamma < 0$  and  $\kappa, \sigma > 0$  - this corresponds to generalized CES preferences, as in [Arkolakis et al. \(2015\)](#);
3.  $\gamma > 0$  and  $\kappa, \sigma < 0$  - this corresponds to Constant Absolute Pass-Through (CAPT).

We focus on cases 2. and 3. The term  $-\kappa/\gamma$  is well defined and positive, and there is a choke price, given by

$$p_j^* = (-\kappa/\gamma)^{1/\sigma}. \quad (20)$$

Log demand can be rewritten as

$$\ln q_j(\omega) = \ln \gamma + \ln(1 - e^{-\sigma \ln x_j(\omega)}), \quad (21)$$

where  $x(\omega) \equiv \frac{p(\omega)}{p^*}$ , as defined in Section 2.1. The price elasticity of demand is

$$\varepsilon_j(\omega) = \sigma \frac{x_j(\omega)^{-\sigma}}{x_j(\omega)^{-\sigma} - 1}. \quad (22)$$

We shall note that since  $x(\omega)$  takes values between 0 and 1,  $x_j(\omega)^{-\sigma} > 1 \iff \sigma > 0$ . This implies that for  $\sigma > 0$ , we get  $\varepsilon_j(\omega) > \sigma$ . Similarly, when  $\sigma < 0$ , we have  $x_j < 1$ , and  $\varepsilon_j(\omega) > \sigma$ . We also notice that the demand elasticity is undoubtedly greater than 1 only if  $\sigma > 1$ , whereas we shall restrict  $x_j > (1 - \sigma)^{\frac{1}{\sigma}}$  whenever  $\sigma < 1$ . To study the sign of the elasticity, we write the “super-elasticity”, or the elasticity of the elasticity, as

$$\frac{\partial \ln \varepsilon_j(\omega)}{\partial \ln x_j(\omega)} = \varepsilon_j(\omega) - \sigma > 0, \quad (23)$$

which, as we discussed above, is always positive regardless of the sign of  $\sigma$ . Therefore, demand elasticity is always increasing in the relative price  $x_j$ . To the extent that the relative price is lower for more productive firms (which is usually implied by standard models), the elasticity is lower for more productive firms. The markup is given by

$$\mu_j(\omega) = \frac{\sigma}{\sigma - 1 + x_j(\omega)^\sigma}, \quad (24)$$

and it is always decreasing in  $x$ , i.e.  $\mu'(x) < 0 \forall \sigma$ . We can now derive markup elasticity as

$$\Gamma_j(\omega) = \frac{\varepsilon_j(\omega) - \sigma}{\varepsilon_j(\omega) - 1}. \quad (25)$$

The sign of the markup elasticity depends on  $\sigma$ . In particular, we note that

$$\Gamma_j(\omega) > 0 \iff \varepsilon_j(\omega) > 1, \quad (26)$$

which is always true if we restrict  $x_j > (1 - \sigma)^{\frac{1}{\sigma}}$  whenever  $\sigma < 1$ . Moreover

$$\frac{\partial \ln \Gamma_j(\omega)}{\partial \ln x_j(\omega)} = \mu_j(\omega) (\sigma - 1) \quad (27)$$

which is positive only if  $\sigma > 1$ . Equations (26) and (27) will be useful in our discussion of pass-through. In particular, the expression for pass-through is given by

$$\Phi_j(\omega) = \frac{1}{1 + \frac{\varepsilon_j(\omega) - \sigma}{\varepsilon_j(\omega) - 1}}. \quad (28)$$

Equation (26) implies that  $\Phi_j(\omega) \in (0, 1)$ , that is pass-through is always incomplete. In order to study how pass-through is related to firm productivity, we shall study the sign of  $\Gamma'$ . In particular,  $\Phi_j(\omega)$  will be lower for more productive firms if  $\Phi' > 0$ , which occurs when  $\Gamma' < 0$ . Equation (27) says that this is true if  $\sigma < 1$ .

Based on these results, we can play with the following cases, all of which have the feature

that  $\mu'(x) < 0$ :

(a)  $\gamma < 0, \sigma > 1$ . This is generalized CES as in [Arkolakis et al. \(2015\)](#), and has pass-through *increasing* in sales.

(b)  $\gamma < 0, \sigma = 1$ . This is Stone Geary, or generalized CES with  $\sigma = 1$ , as in [Simonovska \(2015\)](#), and has pass-through constant in sales.

(c)  $\gamma < 0, \sigma \in ]0, 1[$ . This is also generalized CES, with pass-through decreasing in sales.

(d)  $\gamma > 0, \sigma < 0$ . This is CAPT, with pass-through decreasing in sales.

(e) When  $\sigma \rightarrow 0$ , we get the CARA demand function with  $\gamma > 0$  and  $\kappa < 0$ , i.e.

$$q = \gamma + \kappa \ln p.$$

This is used by [Behrens et al. \(2016\)](#). Since this is between cases (c) and (d) above, it has passthrough decreasing in sales.

## 2.3 Kimball Preferences

One set of preferences that has been often used in the variable markups literature is the one first studied by [Kimball \(1995\)](#). Preferences are described by the implicit function:

$$\int_{\Omega_j} A_j(\omega) Y\left(\frac{q_j(\omega)}{A_j(\omega) Q_j}\right) d\omega = 1, \quad (29)$$

where  $A_j(\omega)$  is a taste parameter, which we can set for simplicity equal to 1. The function  $Y$  is such that  $Y(1) = 1$ ,  $Y'(\cdot) > 0$  and  $Y''(\cdot) < 0$ . We can derive the demand of the consumer in country  $j$  by solving a standard cost minimization problem, taking prices and aggregate quantity  $Q_j$  as given :

$$\begin{aligned} \min_{q_j(\omega)} \int_{\Omega_j} p_j(\omega) q_j(\omega) \\ \text{s.t. (29).} \end{aligned} \quad (30)$$

Let  $\Psi(x) \equiv Y'^{-1}(x)$  denote the inverse of the first derivative of the function  $Y(\cdot)$ . Then

$$q_j(\omega) = \Psi\left(\tilde{Q}_j \frac{p_j(\omega)}{P_j}\right) Q_j, \quad (31)$$

where  $P_j$  is the price index in market  $j$ , defined as  $P_j = \int_{\Omega_j} p_j(\omega) \frac{q_j(\omega)}{Q_j} d\omega$ , and where  $\tilde{Q}_j \equiv \int_{\Omega_j} Y'\left(\frac{q_j(\omega)}{Q_j}\right) \frac{q_j(\omega)}{Q_j} d\omega$ . It is useful to rewrite (31) in log-terms as

$$\log q_j(\omega) = \log \Psi\left(\tilde{Q}_j \frac{p_j(\omega)}{P_j}\right) + \log Q_j \quad (32)$$

**Characterization** To put more structure, we follow [Klenow and Willis \(2006\)](#) and choose a flexible functional form for  $Y(\cdot)$  that is such that<sup>8</sup>

$$\log \Psi(y) = \frac{\theta}{\eta} \log \left[ 1 + \eta \log \left( \frac{\theta - 1}{\theta y} \right) \right], \quad \eta > 0, \theta > 1 \quad (33)$$

where  $y \equiv \tilde{Q}_j \frac{p_j(\omega)}{P_j}$ . Using this functional form assumption and the definition in (10), it can be easily shown that the choke price can be written as

$$p_j^* = \exp\left(\frac{1}{\eta}\right) \frac{\theta - 1}{\theta} \frac{P_j}{\tilde{Q}_j}, \quad (34)$$

which implies that we can rewrite (32) as

$$\log q_j(\omega) = \frac{\theta}{\eta} \log(-\eta \log x_j(\omega)) + \log Q_j \quad (35)$$

From equation (35), demand elasticity can be easily derived as

$$\varepsilon_j(\omega) = -\frac{\theta}{\eta \log x_j(\omega)}. \quad (36)$$

In the CES limit with  $\eta \rightarrow 0$ , it's easy to show that  $\eta \log x \rightarrow -1$  and that  $\varepsilon_j(\omega) \rightarrow \theta$ . For

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<sup>8</sup> $Y(x) = 1 + (\theta - 1) \exp\left(\frac{1}{\eta}\right) \eta^{\left(\frac{\theta}{\eta}\right)-1} \left(\Gamma\left(\frac{\theta}{\eta}, \frac{1}{\eta}\right) - \Gamma\left(\frac{\theta}{\eta}, \frac{x^{\frac{\theta}{\eta}}}{\eta}\right)\right)$ .

$\eta > 0$ ,  $\varepsilon_j(\omega)$  is increasing in  $x$  : more productive firms face lower demand elasticities<sup>9</sup>. The markup is given by

$$\mu_j(\omega) = \frac{\theta}{\theta + \eta \log x_j(\omega)}, \quad (37)$$

and markup elasticity is

$$\Gamma_j(\omega) = \frac{\eta}{\theta + \eta \log x(\omega)} > 0 \text{ for } \eta \log x \in [-1, 0]. \quad (38)$$

It's easy to show that both markup and markup elasticity are decreasing in  $x_j(\omega)$ . This means that more productive firms charge higher markups, and change markups more with a marginal increase in  $x_j(\omega)$ . Finally, we discuss pass-through. Using (17), we can write pass-through as

$$\Phi_j(\omega) = \frac{\theta + \eta \log x(\omega)}{\theta + \eta \log x(\omega) + \eta} > 0. \quad (39)$$

As long as  $\eta > 0$ , this demand always generate incomplete pass-through of cost shocks (i.e.  $\Phi_j(\omega) < 1$ ). Moreover,  $\Phi' > 0$  always, which means that high productivity firms (low  $x_j(\omega)$ ) have lower pass-through of cost-shocks into prices.

## 2.4 AddiLog Preferences

We now consider the symmetric, indirectly additive preferences which has been studied by [Bertoletti et al. \(2015\)](#). Preferences are described by the following additive specification<sup>10</sup>

$$V = \int_{\Omega_j} Y \left( \frac{p_j(\omega)}{I_j} \right) d\omega. \quad (40)$$

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<sup>9</sup>Note that since we want to restrict the elasticity to be always greater than 1 in equilibrium  $\forall \theta$ , we impose that  $x \in [\exp(-\frac{\theta}{\eta}), 1]$ .

<sup>10</sup>With the exception of CES preferences that it encompasses, (40) represents a class of preferences that are neither homothetic nor directly additive.

The indirect utility  $V$  depends on the normalized prices  $\frac{p_j(\omega)}{I_j}$  of each consumed variety  $\omega \in \Omega_j$ , where  $I_j$  is the income. Function  $Y(\cdot)$  is a decreasing and convex function up to a (possibly infinite) choke value  $\alpha$ . The choke value has the property that  $p_j^* = \alpha I_j$  is the reservation (choke) price of each variety, with  $Y(x), Y'(x) = 0$  for all  $x \geq \alpha$ . Demand can be derived using Roy's identity as

$$q_j(\omega) = \frac{Y' \left( \frac{p_j(\omega)}{I_j} \right)}{\tilde{P}_n} \quad (41)$$

where

$$\tilde{P}_n = \int Y' \left( \frac{p_j(\omega)}{I_j} \right) \frac{p_j(\omega)}{I_j} d\omega. \quad (42)$$

Note that the integral in the bottom is over all goods and not only the ones consumed, because as wealth changes individual might want to change the optimal consumption bundle.

**Characterization** We follow [Bertoletti and Etro \(2015\)](#) and write

$$Y \left( \frac{p_j(\omega)}{I_j} \right) = \frac{\left( \alpha - \frac{p_j(\omega)}{I_j} \right)^{1+\gamma}}{1 + \gamma}, \quad (43)$$

where  $\gamma > 0$  and  $\alpha$  is the choke value. The CES is simply given by taken the limit  $\alpha \rightarrow \infty$ .

Using the properties of the choke price, we can rewrite  $I_j = \frac{p_j^*}{\alpha}$ , which implies that

$$\frac{p_j(\omega)}{I_j} = \alpha x_j(\omega). \quad (44)$$

Using (43) and (44) we can write (log) demand as

$$\log q_j(\omega) = \gamma \log(1 - x_j(\omega)) - (1 + \gamma) \log \alpha - \log \tilde{P}_n \quad (45)$$

The elasticity of demand is

$$\varepsilon_j(\omega) = \gamma \frac{x_j(\omega)}{1 - x_j(\omega)}. \quad (46)$$

Note that here  $\gamma$  is the key preference parameter. Demand is linear for  $\gamma = 1$ , tends to become perfectly elastic for  $\gamma \rightarrow \infty$  and perfectly rigid for  $\gamma \rightarrow 0$ . Since we want this elasticity to be greater than one (and demand tends to become perfectly rigid as  $x \rightarrow 0$ ), we impose  $\gamma \frac{x}{1-x} > 1 \iff x > \frac{1}{\gamma+1}$ . The firm markup is given by

$$\mu_j(\omega) = \frac{\gamma x_j(\omega)}{(\gamma + 1)x_j(\omega) - 1}. \quad (47)$$

Asymptotically, as  $x \rightarrow 1$ ,  $\implies \mu \rightarrow 1$  and as  $(\gamma + 1)x_j(\omega) \rightarrow 1 \implies \mu_j \rightarrow \infty$ : markups increase with firm size. The elasticity of the markup is

$$\Gamma_j(\omega) = \frac{1}{(\gamma + 1)x_j(\omega) - 1} > 0 \quad (48)$$

The implied pass-through of shocks into prices stemming from direct effects is given by

$$\Phi_j(\omega) = \frac{(\gamma + 1)x_j(\omega) - 1}{(\gamma + 1)x_j(\omega)}. \quad (49)$$

Equation (49) implies that asymptotically, when  $(\gamma + 1)x \rightarrow 1$ ,  $\Phi_j \rightarrow 0$ . This means that when preferences are Addilog, more productive firms have lower pass-through into their prices.

## 2.5 QMO $\rho$ (Feenstra, 2014)

We now consider [Feenstra \(2014\)](#), and study the following expenditure function over a continuum of goods  $\omega$  :

$$e_\rho = \left[ \alpha \int p_j(\omega)^\rho d\omega + \beta \left( \int p_j(\omega)^{\frac{\rho}{2}} d\omega \right)^2 \right]^{\frac{1}{\rho}}, \quad \rho \neq 0. \quad (50)$$

Equation (50) represents the expenditure needed to obtain one unit of utility <sup>11</sup>. The CES case is equivalent to set  $\alpha > 0$ ,  $\beta = 0$  and  $\rho > 0$ . By Shepard's lemma, the demand of any variety  $\omega$  can be derived as

$$q_j(\omega) = \frac{\partial E_j(u, \mathbf{p})}{\partial p_j(\omega)}, \quad \forall \omega \in \Omega_j, \quad (51)$$

where  $E_j(u, \mathbf{p}) = P_j Q_j \equiv e_\rho(\mathbf{p})u$  is total expenditure, and  $\Omega_j \equiv \{\omega | p_j(\omega) < p_j^*\}$  is the set of goods available for consumption, with mass  $N_j \equiv \int_{\Omega_j} d\omega > 0$ . The mass of all possible goods is denoted instead by  $\tilde{N}_j \equiv \int d\omega > N_j$ . Let  $p_j^*$  be the choke price for any variety  $\omega$ . Goods that are not available (mass  $\tilde{N}_j - N_j$ ) should have their price in (50) replaced by  $p_j^*$ . This price can be derived by setting to zero the demand for goods with price equal to the reservation price, i.e.

$$0 = \frac{\partial E_j(u, \mathbf{p})}{\partial p_j^*}, \quad \forall \omega \notin \Omega_j.$$

This gives

$$p_j^* = \left( \frac{N}{N - [\tilde{N} + (\alpha/\beta)]} \right)^{2/\rho} \left( \int_{\Omega_j} \frac{1}{N} p_j(\omega)^{\rho/2} d\omega \right)^{2/\rho}. \quad (52)$$

Hence, the reservation price  $p_j^*$  can be written as a constant times the ‘‘power mean’’ of prices. To ensure that the reservation price (52) is above the mean price, and to rule out the CES case of an infinite reservation price, we need conditions on the parameters. In particular, we

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<sup>11</sup>Equation (50) can be derived as the limit of a quadratic mean of order  $\rho$  unit cost function, i.e.  $\left[ \sum_i \sum_j b_{ij} p_i^{\frac{\rho}{2}} p_j^{\frac{\rho}{2}} \right]^{\frac{1}{\rho}}$ , where  $\alpha \equiv b_{ii}$  and  $\beta \equiv b_{ij}$ ,  $i \neq j$ , and the number of goods goes to infinity.



posit the following assumption:

- A1        If  $\rho < 0$  then  $\alpha > 0$ ,  $\beta < 0$  and  $[\tilde{N} + (\alpha/\beta)] < 0$ ;
- A2        If  $\rho > 0$  then  $\alpha < 0$ ,  $\beta > 0$  and  $0 < [\tilde{N} + (\alpha/\beta)] < N$ ;
- A3        As  $\rho \rightarrow 0$ , then  $\alpha = \left(\frac{1}{\tilde{N}} - \frac{2\gamma}{r}\right)$  and  $\beta = \frac{2\gamma}{r\tilde{N}}$  for any  $\gamma > 0$ .

Under A1-A3, it can be shown that for  $N \geq 0$  and  $\rho \leq 2$  the QMO $\rho$  expenditure function is globally positive, non-decreasing, homogeneous of degree one and concave in prices, with a finite reservation price. This ensures that it can be derived from a well-behaved homothetic utility function.<sup>12</sup>

**Characterization** As shown by Feenstra (2014), demand can be written as

$$q_j(\omega) = \frac{\alpha u}{\tilde{e}_\rho} x_j(\omega)^{\rho-1} (1 - x_j(\omega)^{-\frac{\rho}{2}}) \quad (53)$$

where  $\tilde{e}_\rho \equiv \left\{ \int_{\Omega_j} \alpha x_j(\omega)^\rho [1 - x_j(\omega)^{-\rho/2}] d\omega \right\}^{\frac{1}{\rho}}$ . Equation (53) shows that demand can be written as a function of the sufficient statistic  $x_j(\omega)$ . In logs, equation (53) becomes

$$\ln q_j(\omega) = \log \frac{\alpha u}{\tilde{e}_\rho} + (\rho - 1) \log x_j(\omega) + \log \left( 1 - \left( \exp -\frac{\rho}{2} \log x_j(\omega) \right) \right), \quad (54)$$

We directly evaluate the elasticity of demand by differentiating (54) with respect to  $x_j(\omega) < 1$ , holding utility  $u$  and  $\tilde{e}_\rho(\mathbf{p})$  constant. This gives

$$\varepsilon_j(\omega) = 1 - \rho - \frac{\rho}{2} \left( \frac{1}{x_j(\omega)^{\frac{\rho}{2}} - 1} \right). \quad (55)$$

Note that as long as  $\beta \neq 0$  (55) is always positive under A1-A3. A sufficient condition for  $\varepsilon_j(\omega) > 1$  is that  $\rho \leq 0$ ,<sup>13</sup> which we will assume in the following discussion. In the limit, for

<sup>12</sup>For values of  $\rho > 2$ , the demand curves in (51) are still downward sloping in a neighborhood of the reservation price, but we cannot guarantee this property globally.

In order to derive demand, we can substitute (52) in (50), set  $p(\omega') = p$  in a small interval  $\omega' = [\omega, \omega + \epsilon]$  and apply Shepard's lemma as before.

<sup>13</sup>For  $0 < \rho \leq 2$ ,  $\eta(\omega) \geq 0$ .

$x_j(\omega) \rightarrow 0$ , elasticity goes to  $\varepsilon_j(\omega) \rightarrow 1 - \rho$ . On the contrary, as  $x_j(\omega) \rightarrow 1$ ,  $\varepsilon_j(\omega) \rightarrow \infty$ , which means that the elasticity is increasing in the relative price. The markup function is given by

$$\mu_j(\omega) = 1 - \frac{1}{\rho} \left( \frac{2x_j(\omega)^{\frac{\rho}{2}} - 2}{2x_j(\omega)^{\frac{\rho}{2}} - 1} \right) \quad (56)$$

which is such that  $\mu' < 0$  for  $x_j(\omega) \in [0, 1]$ . We derive the elasticity of the markup as

$$\Gamma_j = \frac{x_j(\omega)^{\frac{\rho}{2}}}{(1 - 2x_j(\omega)^{\frac{\rho}{2}})^2 \mu_j(\omega)}, \quad (57)$$

which is always positive. It can be shown that  $\Gamma' > 0$ , which means that markups are more sensitive to changes in relative prices the higher the price, i.e. the less productive the firms. Finally, we discuss pass-through. Using our formula for direct pass-through of shocks into prices we find that in the case of QMO $\rho$  preferences,  $\Phi_j(\omega)$  is given by

$$\Phi_j(\omega) = \frac{(2x_j^{\frac{\rho}{2}} - 1) \left( (\rho - 1)x_j^{\frac{\rho}{2}} + 1 - \frac{\rho}{2} \right)}{2\rho x_j(\omega)^\rho - 2x_j(\omega)^\rho - \frac{3}{2}\rho x_j(\omega)^{\frac{\rho}{2}} + 3x_j(\omega)^{\frac{\rho}{2}} + \frac{\rho}{2} - 1} \quad (58)$$

and given that  $\Gamma' > 0$ , we know that  $\Phi' < 0$  which means that pass-through decreases with the relative price  $x_j(\omega)$ , or increase with firm productivity.

## 2.6 Summary

Table 1 summarizes the theoretical results derived so far. For each of the demand function considered, it reports how elasticity of demand, markup, markup elasticity and pass-through are related to the sufficient statistics  $x_j(\omega)$ .

Table 1: SUMMARY OF RESULTS

PREFERENCES	POLLAK		KIMBALL		ADDILOG		QMO $\rho$	
	$\varepsilon(x)$	$\sigma \frac{x^{-\sigma}}{x^{-\sigma}-1}$	$\varepsilon' > 0$	$-\frac{\theta}{\eta \log x}$	$\varepsilon' > 0$	$\gamma \frac{x}{1-x}$	$\varepsilon' > 0$	$1 - \rho - \frac{\rho}{2} \left( \frac{1}{x^{\frac{\rho}{2}-1} \right)$
$\mu(x)$	$\frac{\sigma x^{-\sigma}}{(\sigma-1)x^{-\sigma}+1}$	$\mu' < 0$	$\frac{\theta}{\theta+\eta \log x}$	$\mu' < 0$	$\frac{\gamma x}{(\gamma+1)x-1}$	$\mu' < 0$	$1 - \frac{1}{\rho} \left( \frac{2x^{\frac{\rho}{2}-2}}{2x^{\frac{\rho}{2}-1} \right)$	$\mu' < 0$
		$\Gamma(x)$	$\frac{\varepsilon(x)-\sigma}{\varepsilon(x)-1}$	$\frac{\eta}{\theta+\eta \log x}$	$\Gamma' < 0$	$\frac{1}{(\gamma+1)x-1}$	$\Gamma' < 0$	$\frac{x_j(\omega)^{\frac{\rho}{2}}}{(1-2x_j(\omega)^{\frac{\rho}{2}})^2} \mu_j(\omega)$
$\Phi(x)$	$\frac{1}{1+\frac{\varepsilon(x)-\sigma}{\varepsilon(x)-1}}$	$\Phi' \lesseqgtr 0$	$\frac{\theta+\eta \log x}{\theta+\eta \log x+\eta}$	$\Phi' > 0$	$\frac{(\gamma+1)x-1}{(\gamma+1)x}$	$\Phi' > 0$	$\frac{1}{1+\Gamma(x)}$	$\Phi' < 0$
SUFFICIENT STATISTIC $x = \frac{p_j(\omega)}{p_j^*}$								
COMMENTS	$\Gamma' < 0 \iff \Phi' > 0 \iff \sigma < 1$	$\varepsilon > 1 \iff \log x > -\frac{\theta}{\eta}$	$\varepsilon > 1 \iff \log x > -\frac{\theta}{\eta}$	$\varepsilon > 1 \iff x > \frac{1}{1+\gamma}$	$\rho \leq 0 \implies \varepsilon_j(\omega) > 1$			

### 3 Analysis

So far we have not imposed any assumption on the GE environment. In fact, the results summarized in Table 1 depend exclusively on the properties of the demand functions, regardless of firm entry and costs. We now go one step further, and integrate the preferences described in (1) and (2) in a standard setup in international trade (e.g. Melitz (2003)). We show that in this model, our sufficient statistics  $x_j(\omega)$  can be mapped one-to-one to firm's productivity. We exploit this nice feature of the model to derive the markup and pass-through distribution in the economy, and study the empirical relevance of the different demand functions.

#### 3.1 Mapping sufficient statistic to firm efficiency

We assume free entry in each destination market. We assume that production of each variety uses only labor inputs, and that in each country firms differ by their level of productivity  $z$ , which they draw upon entry from some known distribution. Firms can be indexed by either  $\omega$ , or their productivity  $z$ . The marginal cost of the firm producing  $\omega$  is

$$c_j(\omega) = \frac{\vartheta_j}{z}, \tag{59}$$

where  $\vartheta_j$  is the component of costs that is common across varieties, which might include wages, iceberg transport costs and so on. Let  $z_j^*$  be the productivity of the least efficient firm active in market  $j$ . The assumption of free entry implies that in each market  $j$ , the firm with productivity  $z = z_j^*$  will make zero profits, and in particular it will charge the competitive price, i.e.

$$p_j^* = c_j(\omega^*) = \frac{\vartheta_j}{z_j^*}. \tag{60}$$

Given that markups can be written as a function of the relative price  $x_j(\omega) \equiv \frac{p_j(\omega)}{p_j^*}$ , i.e.

$\mu_j(\omega) = \mu(x_j(\omega))$ , the equilibrium price of  $\omega$  in country  $j$  can be expressed as

$$p_j(\omega) = \mu(x_j(\omega)) c_j(\omega) \quad (61)$$

We can substitute (59) and (60) in (61), rearrange and write

$$\frac{z}{z_j^*} = \frac{\mu(x_j(\omega))}{\underbrace{x_j(\omega)}_{\equiv \phi(x_j(\omega))}}. \quad (62)$$

Note that  $\phi'(x) = \frac{\mu'x - \mu}{x^2} < 0 \forall x$ , which means that  $\phi(x)$  is monotonic, and can be inverted to write

$$x_j(\omega) = \phi^{-1}\left(\frac{z}{z_j^*}\right). \quad (63)$$

Equation (63) shows how we can map our sufficient statistic  $x$  to the firm rank  $\frac{z}{z^*}$ , and in particular, it can be easily shown that with  $x_j$  decreasing in  $\frac{z}{z^*}$ <sup>14</sup>. In this setting, more efficient firms have lower marginal costs, face lower elasticity of demand, and charge higher markups and lower prices<sup>15</sup>. Note that (63) implies that we could rewrite all the measures of firm performance summarized in Table 1 in terms of the firm's rank in the relevant market. Since it is not possible to get closed form expressions in this case, we solve for this expressions numerically. For simplicity, we choose values for parameters according to what has been done in other works<sup>16</sup>. In Figure 1a. and 1b., we show how markups and pass-through depends

<sup>14</sup>To sign equation (63), note that by the properties of the derivative of an inverse function

$$(\phi^{-1})'(x) = \frac{1}{\phi'(\phi(x))}, \quad (64)$$

which says that  $(\phi^{-1})'$  and  $\phi'$  have the same sign.

<sup>15</sup>Since  $\varepsilon_j(\omega) > 1$ , the markup will increase less than linearly in  $x$ , which means that more productive firms will charge lower prices in equilibrium (i.e. the cost effect prevails over the markup effect).

<sup>16</sup>The following table summarizes our choice

	POLLAK	KIMBALL	ADDILOG	QMO $\rho$
PARAMETERS	$\gamma = 1.5; \sigma = 5$	$\theta = 5; \eta = 4$	$\gamma = 1.4;$	$\rho = -5$
SOURCE	<a href="#">ARKOLAKIS ET AL. (2015)</a>	<a href="#">GOPINATH ET AL. (2010)</a>	<a href="#">BERTOLETTI ET AL. (2015)</a>	-

FIGURE 1A. MARKUPS AND FIRM SIZE

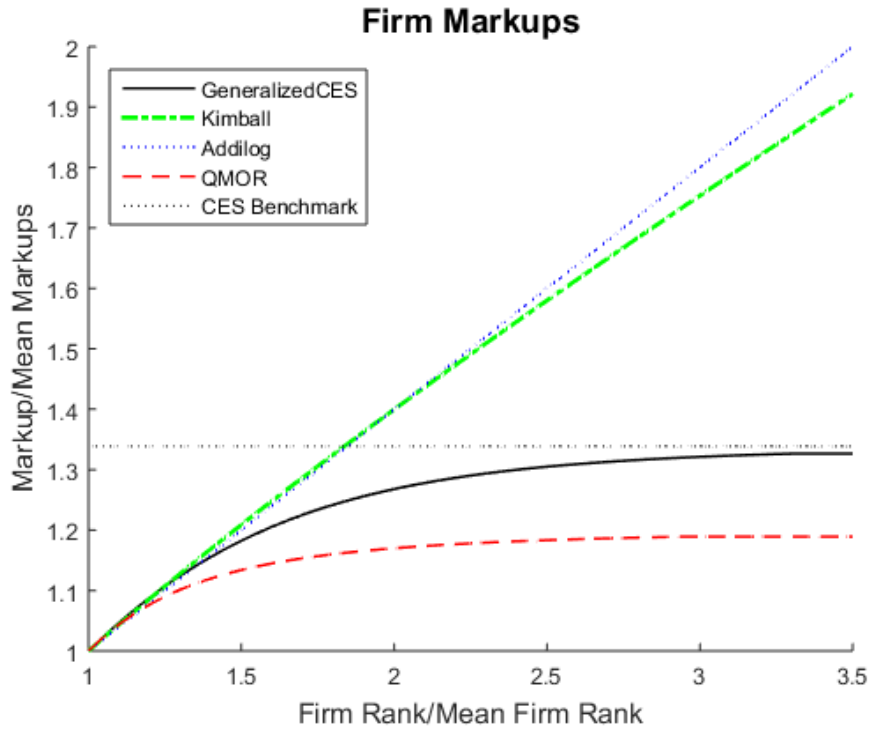
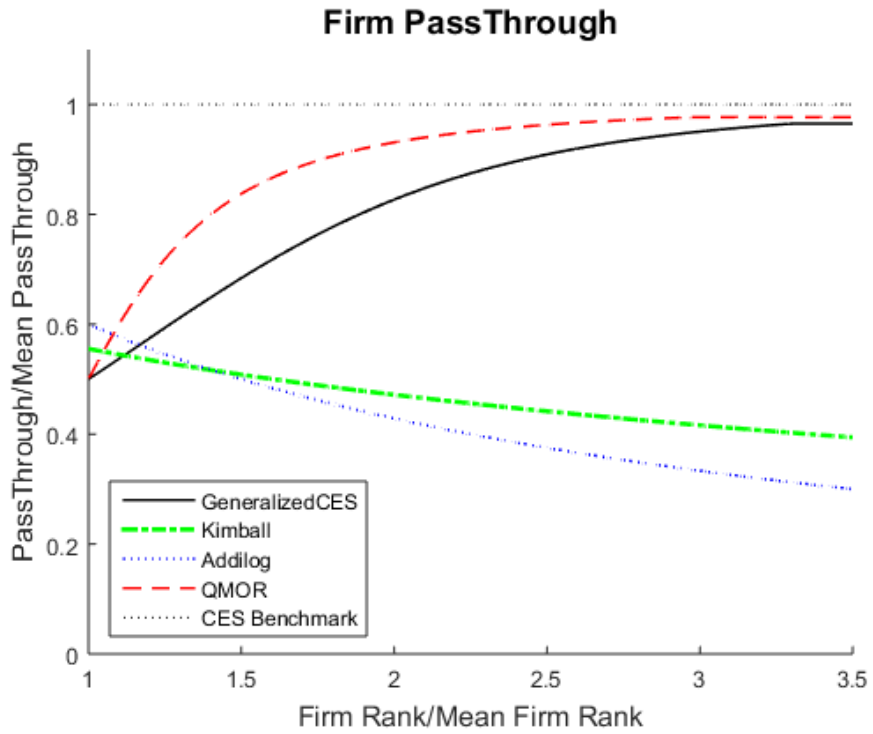


FIGURE 1B. PASS-THROUGH AND FIRM SIZE



on firm’s rank, respectively. As noted above, we want to remark that both the Generalized CES and the QMOR demand functions imply that pass-through increases with firm size, a fact which is at odds with the evidence on variable markups and firm’s pass-through. We review this and other major results in the empirical literature in the next paragraph. This will be useful for our next exercise in Section 3.3, where we calibrate a standard size distribution of firms and derive the implied markups and pass-through distribution in the simulated economy.

### **3.2 Empirical Evidence on Markups and Exchange-Rate Pass-Through**

The pass-through literature recognizes both that prices and demand elasticities vary across firms, and that firms respond only partially to cost shocks, which in turn generates price and markups dispersion across firms. Representative papers include [Goldberg and Verboven \(2001\)](#), [Nakamura and Zerom \(2010\)](#), [Berman et al. \(2012\)](#), [Goldberg and Hellerstein \(2013\)](#), and [Amiti et al. \(2014\)](#). Although the pass-through literature focus almost exclusively on exchange rate shocks, its insights are equally applicable to any exogenous cost-push shock. Most of the pass-through literature uses a structural approach where assumptions on the demand side, market structure, and firm behavior are combined to derive measures of firm performance. An alternative approach is to estimate markups from the the so-called “production function approach”, which is now quite common in the industrial organization literature. The key insight of using a production function framework is that under imperfect competition, input growth is associated with disproportional output growth, as measured by the relevant markup. Relevant papers are [Hall \(1988\)](#), [Klette \(1999\)](#), and more recently [De Loecker and Goldberg \(2014\)](#); [De Loecker et al. \(2016\)](#). Interestingly, the insights obtained using the two different approaches turn out to be consistent with each other. In particular it has been found that:

**F1:** *There is substantial heterogeneity across firms, whereby more productive firms charges higher markups than low productivity firms.*

Similarly, exporters seem to charge higher markups than importers (Amiti et al. (2014)), and these difference are both statistically and economically significant.

**F2:** *Exchange rate pass-through is incomplete, and is lower for high productivity firms than for low productivity firms.*

The incompleteness of exchange rate pass-through is perhaps the most robust finding in the literature on international prices and shocks, and it is true both at the individual and aggregate level (see Goldberg and Knetter (1997) for a survey). The heterogeneity of pass-through across firms is a more recent finding, and was first documented by Berman et al. (2012), and more recently by Amiti et al. (2014) and Chatterjee et al. (2013). In particular, these papers find that more productive firms have lower pass-through into their prices because they adjust markups more after an exchange rate shock. This suggest that their markup elasticity is higher than that of low productivity firms. We saw how pass-through and markup elasticity are very closely related. Therefore, variable markups has been acknowledged as one of the main drivers of incomplete pass-through (Goldberg and Hellerstein (2013)).

### 3.3 A Simple Calibration Exercise

We now assume that firm-level productivity is randomly drawn from a Pareto distribution<sup>17</sup>. We plot the corresponding implied markups and pass-through distribution in Figure 2a and 2b, respectively, as normalized by mean markup and pass-through. Consistently with the empirical fact F1, we do observe heterogeneous markups across the firm size distribution, which are increasing in firm productivity. In this sense, all the demand functions do well. The

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<sup>17</sup>As argued by Arkolakis et al. (2012), this is probably the most common restriction on technology imposed in the literature on monopolistic competition with firm heterogeneity emanating from Melitz (2003), for both empirical and theoretical reasons. We use a Pareto elasticity of  $\theta = 5$ , as calibrated by Arkolakis et al. (2015) using data for French firms.



FIGURE 2A. MARKUP DISTRIBUTION

**Firm Markup by Percentile**

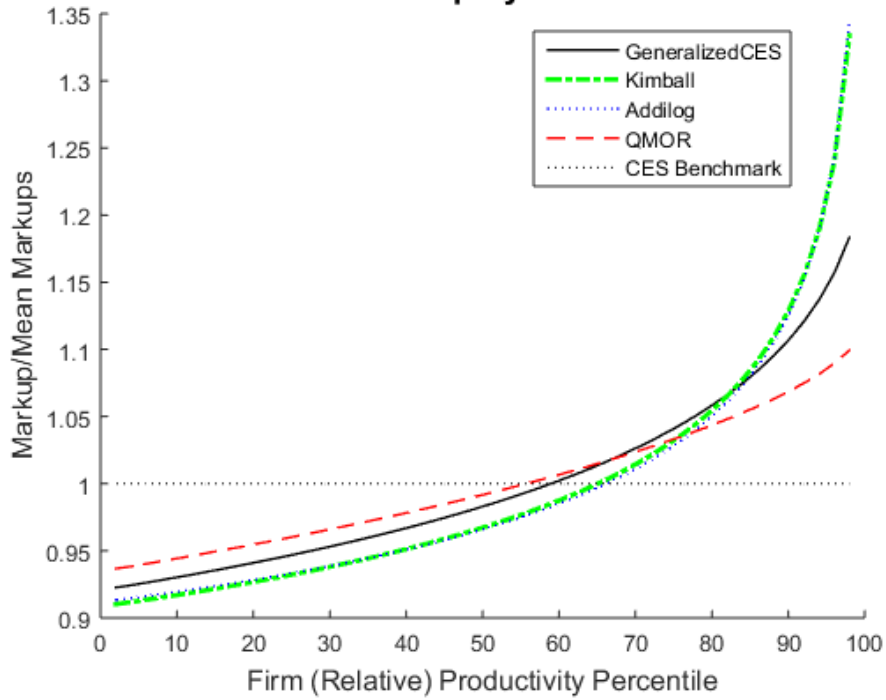
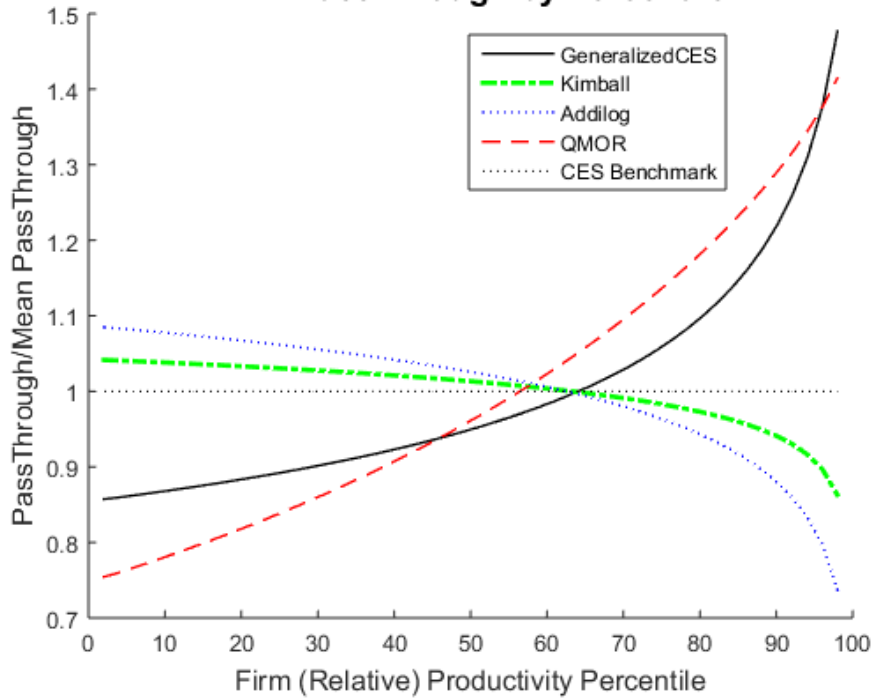


FIGURE 2B. PASS-THROUGH DISTRIBUTION

**Firm PassThrough by Percentile**



Addilog and Kimball demand allow quantitatively for greater variation in markups across firms. For example, a firm at the 95th percentile would charge a markup that is almost 30% higher than the median firm if demand is Kimball or Addilog, and only 15% higher than the median firm if demand is QMOR or Generalized CES. This suggest that Kimball or Addilog may perform better in matching the high degree of markup variability observed in the data. Let's now move to pass-through. We notice from Figure 2 that all the demand functions generate incomplete pass-through. Again, we find that both Kimball and Addilog are the most consistent with the empirical evidence, since they also generate pass-through decreasing in firm productivity. Both Generalized CES and QMOR instead have the undesired feature that high productivity firms have higher pass-through than low productivity firms.

### **3.4 Empirical Challenges of the non-CES+MC model**

We showed that by relaxing the CES assumption, one can go a long way in improving the model fit of the data. In this paragraph we discuss some limitations of the non-CES+MC approach that have been highlighted in the literature. In particular, [Amiti et al. \(2016\)](#) argued that such framework is not well suited for matching their evidence on strategic complementarities in pricing across firms. The authors use an oligopolistic CES model instead. In the next section, we review the main papers in the literature of oligopolistic competition and discuss further theoretical and empirical properties.

## **4 Other Models of Variable Markups**

We now consider alternative approaches that have been used in the literature to generate variable markups. We start by reviewing the [Atkeson and Burstein \(2008\)](#) model, where CES demand is combined with a discrete number of firms (products). In the model, more productive firms have higher market shares, higher markups and lower pass-through than low productivity firms, and variable markups are achieved by means of strategic firm be-

havior in oligopolistic competition. We then discuss two relatively simple departures from the CES oligopolistic competition benchmark. The first model assumes Bertrand rather than Cournot competition among firms, whereas the second model assumes non-CES demand structure along with Cournot competition among firms. The theoretical results are qualitatively unchanged. The last part of the Section discusses a model with distribution costs, which generalizes the model used by [Corsetti and Dedola \(2005\)](#). Although the model does not fit directly in any of the frameworks above, it can still generate desirable qualitative results.

#### 4.1 A Model of Cournot Competition and CES Demand

Individual good varieties are aggregated into sectoral outputs, which are the inputs in the production of a final consumption bundle. Each firm produces a distinct good in a specific sector, and there are only a relatively small number of firms in each sector. Final consumption,  $Q_j$ , is assembled according the following CES aggregator

$$Q_j = \left( \int_0^1 y_{js}^{\frac{\eta-1}{\eta}} ds \right)^{\eta/(\eta-1)}. \quad (65)$$

We assume, for simplicity, that the  $2K$  firms selling in equilibrium is given exogenously<sup>18</sup>. Output in each sector is given by

$$y_{js} = \left[ \sum_{k=1}^{2K} (q_{js}(k))^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, \quad (66)$$

where  $q_{js}(k)$  denotes sales in country  $j$  of firm  $k$  in sector  $s$ . It is assumed that (i) goods are imperfect substitutes:  $\rho < \infty$ ; (ii) goods are better substitutes within sectors than across sectors:  $1 < \eta < \rho$ , and the firms play the following static game:

A. Cournot Firms play a static game of quantity (Cournot) competition. Specifically, each

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<sup>18</sup>This assumption can be easily relaxed, as it is done in the paper, but studying the full model in detail is beyond our scope.

firm  $k$  chooses its quantity  $q_{js}(k)$  sold in country  $j$  to maximize profits, taking as given the quantities chosen by the other firms, as well as the wage rate, and aggregate variables  $P_j$  and  $Q_j$ .

Profits maximization of firms subject to (65) and (66) implies

$$p_{js}(k) = \frac{\varepsilon(s_{js}(k))}{\varepsilon(s_{js}(k)) - 1} MC_{jk}, \quad (67)$$

where

$$\varepsilon(s) = \left[ \frac{1}{\rho}(1-s) + \frac{1}{\eta}s \right]^{-1} \quad (68)$$

is the price elasticity of demand, and  $s_{js}$  is defined as

$$s_{js}(k) = \frac{p_{js}(k)q_{js}(k)}{\sum_{l=1}^{K+K^*} p_{js}(l)q_{js}(l)}. \quad (69)$$

The elasticity of demand is the harmonic average of the elasticity of substitutions  $\rho$  and  $\eta$ , weighted by the market share of firm  $k$  in sector  $s$ ,  $s_{js}(k)$ . Given A1, a firm with a high market share, i.e. with  $s \rightarrow 1$ , is such that  $\varepsilon \rightarrow \eta$  whereas as  $s \rightarrow 0$ , i.e. for low productivity firms,  $\varepsilon \rightarrow \sigma$ . Therefore, elasticity is decreasing in market share, or decreasing in firm productivity. The optimal markup is

$$\mu_{js}(k) = \frac{1}{1 - \left( \frac{1}{\rho}(1 - s_{js}(k)) + \frac{1}{\eta}s_{js}(k) \right)}, \quad (70)$$

increasing in market share. It can be shown that the elasticity of the markup with respect to relative price is

$$\Gamma_{js}(k) = \frac{\partial \mu_{js}}{\partial \log s_{js}} = \frac{s_{js}}{\rho\eta - \eta(1 - s_{js}) - \rho s_{js}} (\rho - \eta) (\rho - 1). \quad (71)$$

Under A1-A2,  $\Gamma_j(s) > 0$ , and it is increasing in market share. In other words, firms with lower relative (log) price  $p_{jsk} - p_{js}$  and higher market share  $s_{js}$  set higher markups, and

change their markups more against shocks to their costs.

#### 4.1.1 Bertrand Competition and CES Demand

Although the original [Atkeson and Burstein \(2008\)](#) paper feature Cournot competition among firms within sectors, results would not qualitatively change if we replaced A.Cournot with

A.Bertrand Firms play a static game of price (Bertrand) competition. Each firm  $k$  chooses  $p_{js}(k)$  to maximize profits, taking as given the price chosen by the other firms, as well as the wage rate, and aggregate variables  $P_j$  and  $Q_j$ .

The elasticity of demand in this case becomes<sup>19</sup>

$$\varepsilon(s) = \rho(1 - s) + \eta s. \quad (72)$$

Unlike(68), the price elasticity of demand under Bertrand competition is a simple average of  $\eta$  and  $\rho$ , with weights given by the market shares. As before, the elasticity is decreasing in  $s_{js}$ , which means that the markups will be increasing in  $s_{js}(k)$ . The elasticity of the markup with respect to relative price is

$$\Gamma_{js}(k) = \frac{s_{js}(1 - s_{js})}{\varepsilon(s_{js})(\varepsilon(s_{js}) - 1)} (\rho - \eta) (\rho - 1), \quad (73)$$

which is always increasing in  $s_{js}$  if  $s_{js} \in [0, \frac{1}{2}]$ . Therefore, pass-through is higher for “smaller” firms. We conclude that the Cournot and Bertrand oligopolistic competition models are qualitatively very similar.

## 4.2 Cournot Competition and non-CES Demand

[Amiti et al. \(2016\)](#) consider an extension of the [Atkeson and Burstein \(2008\)](#) models, featuring oligopolistic competition and Kimball Demand together. Therefore, they generate

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<sup>19</sup>For derivations, see [Amiti et al. \(2016\)](#)

variable markups from the two different channels that we have seen before: both a variable demand elasticity and strategic behavior across firms. Demand in each sector is given by (29), where  $Q$  is sectoral consumption, and  $q(\omega)$  is optimal consumption of good  $\omega$ , within a sector. Consumers allocate expenditure  $E$  to purchase products in the sectors, such as

$$\sum_{\omega \in \Omega} p(\omega)q(\omega) = E. \quad (74)$$

Given prices  $\{p(\omega)\}_{\omega \in \Omega}$  and  $E$ , consumers optimally allocate consumption  $\{q(\omega)\}_{\omega \in \Omega}$  to maximize the consumption index  $Q$  :

$$\max_{\{q(\omega)\}_{\omega \in \Omega}} \{Q \mid \text{s.t. (29) and (74)}\}. \quad (75)$$

The elasticity of demand in this case is given by

$$\Sigma(\omega) \equiv -\frac{\partial \log q(\omega)}{\partial \log p(\omega)} = \eta s(\omega) + \sigma(\omega) \left( 1 - \frac{s(\omega)\sigma(\omega)}{\sum_{\omega' \in \Omega} \sigma(\omega')} \right), \quad (76)$$

where

$$\sigma(\omega) = -\frac{\partial \log \psi(x)}{\partial \log x}, \quad (77)$$

where  $\psi(\cdot) \equiv \Upsilon'^{-1}(\cdot)$  is the demand curve, and  $x$  is the effective price of the firm, given the aggregate variables in equilibrium. The key insight is that the market share channel in (76) operates exactly in the same way as in (68). In the CES model, this is the only channel of markup adjustment, as we have seen before. More generally, with non-CES demand, the markup elasticity also depends on the properties of the  $\sigma(\cdot)$  function in (77), and markups are non-constant even in the limiting case of monopolistic competition with  $s \rightarrow 0$ .

### 4.3 A Model with Distribution Costs

Consider the model of pricing with CES demand and additive distribution costs studied by [Corsetti and Dedola \(2005\)](#). As shown by [Burstein and Gopinath \(2014\)](#), the model gives rise to variable markups of the form assumed above, with demand given by

$$q_j(\omega) = \left( \frac{p_j(\omega)}{P_j} + \delta \right)^{-\sigma} Q_j, \quad (78)$$

where  $\delta > 0$  denotes the fixed distribution cost per good. We now show that the demand function in (78), can be derived from a well-defined indirect utility function. Consider the following indirect utility function

$$V = - \int_{\Omega} \frac{P_j \left( \frac{p_j(\omega)}{P_j} + \delta \right)^{1-\sigma}}{I_j (1-\sigma)} d\omega, \quad (79)$$

with  $\frac{p_j(\omega)}{P_j} + \delta > 0$  and  $\sigma > 1$ . It's easy to show that  $V_{\omega} < 0 \forall \omega$  and that  $V_I > 0$ , is homogeneous of degree zero and strictly quasi-convex. Therefore  $V$  is a valid utility function. We can apply Roy's identity to (79) and get

$$q_j(\omega) = I_j \left( \frac{p_j(\omega)}{P_j} + \delta \right)^{-\sigma}, \quad (80)$$

which is the same as in (78), where I define  $P_j$  implicitly by normalizing  $\int_{\Omega} \frac{\left( \frac{p_j(\omega)}{P_j} + \delta \right)^{1-\sigma}}{1-\sigma} d\omega = 1$ . Unlike the demand functions studied in the main part of the text, in this case preferences do not admit a choke price  $p^*$ . In fact, demand goes to 0 as  $\frac{p_j(\omega)}{P_j} \rightarrow \infty$ , as in the standard CES case. Notice also the when  $\delta < 0$ , price must be such that  $p_j(\omega) > \underline{p}_j$ , where  $\underline{p}_j = \delta P_j$ . In this case, only firms with a price higher than a cutoff level find it profitable to sell in the market. Although we can't rewrite the demand in terms of our sufficient statistic, we can

still characterize its main properties as before. Let us first rewrite (80) in logs as

$$\log q_j(\omega) = \log I_j - \sigma \log \left( \exp \log \left( \frac{p_j(\omega)}{P_j} \right) + \delta \right). \quad (81)$$

We can derive demand elasticity as

$$\varepsilon_j(\omega) = \sigma \frac{y_j(\omega)}{y_j(\omega) + \delta}, \quad (82)$$

where  $y_j(\omega) \equiv \frac{p_j(\omega)}{P_j}$ . Remember that by construction, we need to ensure  $y_j(\omega) > -\delta$ . While this is always true for  $\delta > 0$ , for values of  $\delta < 0$  this means that firms must charge a price that is at least as big as  $-\delta$  to be able to sell in the market. Let's consider the case  $\delta > 0$  first. To ensure that  $\varepsilon > 1$ , we must impose  $y_j > \frac{\delta}{\sigma-1}$ . As the firm gets more productive, the relative price goes to  $y_j \rightarrow \frac{\delta}{\sigma-1}$ , which means  $\varepsilon_j(\omega) \rightarrow 1$ . Conversely, for the low productivity firms we get  $\varepsilon_j(\omega) \rightarrow \sigma$ . Therefore, elasticity is decreasing in productivity. Let's now consider the case  $\delta < 0$ . Remember that in this case,  $y \in (-\delta, \infty)$ . We have  $\varepsilon_j(\omega) \rightarrow \sigma$  for the low productivity firms (i.e. those such that  $y \rightarrow \infty$ ), and  $\varepsilon \rightarrow +\infty$  for the high productivity firms, which means that elasticity increases with firm efficiency, a fact that is at odds with the data. Therefore, we shall focus on the cases such that  $\delta > 0$ , which is the same as in the distribution cost model. We can proceed as it is standard, and derive markups as

$$\mu_j(\omega) = \frac{\sigma y_j(\omega)}{(\sigma - 1)y_j(\omega) - \delta}, \quad (83)$$

which are increasing in firm productivity and go to 1 for the least productive firms. Markup elasticity is given by

$$\Gamma_j(\omega) = \frac{\sigma \delta}{\delta - (\sigma - 1)y_j(\omega)} > 0, \quad (84)$$



and also  $\Gamma' = \frac{\delta(\sigma-1)}{(\delta-\sigma y+y)^2} > 0$ , which means that markup elasticity is increasing in firm productivity. Finally, we derive an expression for pass-through as

$$\Phi_j(\omega) = \frac{(\sigma - 1)y_j(\omega) - \delta}{(\sigma - 1)(y_j(\omega) + \delta)}. \quad (85)$$

We get that pass-through is complete for low productivity firms, i.e. as  $y_j(\omega) \rightarrow 1$ , whereas it goes to 0 as  $y_j(\omega) \rightarrow \frac{\delta}{\sigma-1}$ . Therefore, the demand function in (78) behaves qualitatively as most of the other demand functions considered in this text, and in particular has implications which can be reconciled with the empirical evidence.

## 5 Conclusion

In these notes we studied the properties of some demand functions with variable elasticity of demand. We derived analytical solutions for markups and pass-through of firms for each of these demand functions, and showed how these measures of firm performance can be written as a function of a single sufficient statistic, namely the relative price of the firm to the choke price in the market where the firm operates. We showed that in a standard model of international trade, this sufficient statistic can be mapped one-to-one to firm rank. Under standard distributional assumptions for firm productivity, we derived the markup and pass-through distribution for the simulated economy. We reviewed some of the relevant evidence on variable markups and exchange rate pass-through, and analyzed the qualitative performance of the different demand system. We finally reviewed other models of variable markups, which do not directly fit into the general framework but are nonetheless qualitatively and quantitatively relevant.

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